Lineare Algebra KW 45

Nina Gassner

feedback

this was me correcting your week-5-assignments:



SUCH beautiful proofs
& creative ideas !!! :D

you made HUUGE progress in writing
& coming up with proofs !!!

feedback

Can you find the mistake?

Behauptung: Zwei linear unabhängige Vektoren sind verschieden.

Beweis durch Widerspruch:
Seien $u, v \in \mathbb{R}^n$ linear unabhängig.

Angenommen, $u = v$.
$u = v$
$\Leftrightarrow u - v = 0$
$\Leftrightarrow u, v$ sind linear abhängig.
Widerspruch zur Voraussetzung! □

feedback

Can you find the mistake?

Behauptung: Zwei linear unabhängige Vektoren sind verschieden.

Beweis durch Widerspruch:
Seien $u, v \in \mathbb{R}^n$ linear unabhängig.

Angenommen, $u = v$.
$u = v$
$\Leftrightarrow u - v = 0$
$\Leftrightarrow u, v$ sind linear abhängig.
Widerspruch zur Voraussetzung! □

Vektorräume

quick recap !! :D

–> linalg.guru

in-class

1. Solving linear systems (in-class) (★☆☆)

Consider the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

a) Determine the set of solutions $\mathbf{Sol}(A, \mathbf{b}) = \{\mathbf{x} \in \mathbb{R}^4 : A\mathbf{x} = \mathbf{b}\}$, i.e., write down an *explicit* characterization of this set of solutions in the form presented after Theorem 4.42 on page 151 in the lecture notes.

in-class

$$\left[\begin{array}{rrrr|r} -1 & 2 & 5 & -2 & -6 \\ -3 & 3 & 12 & -3 & -15 \\ 1 & -14 & -7 & -6 & 8 \end{array}\right]$$

in-class

$$\left[\begin{array}{rrrr|r} -1 & 2 & 5 & -2 & -6 \\ -3 & 3 & 12 & -3 & -15 \\ 1 & -14 & -7 & -6 & 8 \end{array}\right] \xrightarrow[\text{III}\leftarrow\text{III}-\text{I}]{\text{I}\leftarrow-\text{I}\\ \text{II}\leftarrow\text{II}+3\text{I}} \left[\begin{array}{rrrr|r} 1 & -2 & -5 & 2 & 6 \\ 0 & -3 & -3 & 3 & 3 \\ 0 & -12 & -2 & -8 & 2 \end{array}\right]$$

in-class

$$\left[\begin{array}{cccc|c} 1 & -2 & -5 & 2 & 6 \\ 0 & -3 & -3 & 3 & 3 \\ 0 & -12 & -2 & -8 & 2 \end{array}\right] \xrightarrow[\substack{\text{I}\leftarrow\text{I}+2\text{II} \\ \text{III}\leftarrow\text{III}+12\text{II}}]{\text{II}\leftarrow-\frac{1}{3}\text{II}} \left[\begin{array}{cccc|c} 1 & 0 & -3 & 0 & 4 \\ 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 10 & -20 & -10 \end{array}\right]$$

in-class

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 0 & 4 \\ 0 & 1 & 1 & -1 & -1 \\ 0 & 0 & 10 & -20 & -10 \end{array}\right] \xrightarrow[\substack{\text{I}\leftarrow\text{I}+3\text{III} \\ \text{II}\leftarrow\text{II}-\text{III}}]{\text{III}\leftarrow\frac{1}{10}\text{III}} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -6 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 \end{array}\right]$$

in-class

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -6 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 \end{array}\right]$$

in-class

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -6 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 \end{array}\right]$$

$$\begin{aligned} x_1 &= 1 + 6t \\ x_2 &= -t \\ x_3 &= -1 + 2t \\ x_4 &= t \end{aligned}$$

in-class

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -6 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 \end{array}\right]$$

$$\begin{aligned} x_1 &= 1 + 6t \\ x_2 &= -t \\ x_3 &= -1 + 2t \\ x_4 &= t \end{aligned}$$

in-class

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -6 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 \end{array}\right]$$

$$\begin{aligned} x_1 &= 1 + 6t \\ x_2 &= -t \\ x_3 &= -1 + 2t \\ x_4 &= t \end{aligned}$$

$$\mathrm{Sol}(A,b) = \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 6 \\ -1 \\ 2 \\ 1 \end{pmatrix} : t \in \mathbb{R} \right\}$$

in-class

1. **Solving linear systems (in-class)** (★☆☆)

   Consider the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

b) Write down a basis for $\mathbf{N}(A)$ (you might have already found it in the previous subtask), and also find a basis for $\mathbf{C}(A)$.



in-class

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -6 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 \end{array}\right]$$

$$\begin{aligned} x_1 &= 1 + 6t \\ x_2 &= -t \\ x_3 &= -1 + 2t \\ x_4 &= t \end{aligned}$$

$$\mathrm{Sol}(A,b) = \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 6 \\ -1 \\ 2 \\ 1 \end{pmatrix} : t \in \mathbb{R} \right\}$$

in-class

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -6 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 \end{array}\right]$$

$$\mathrm{Sol}(A,b) = \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 6 \\ -1 \\ 2 \\ 1 \end{pmatrix} : t \in \mathbb{R} \right\}$$

in-class

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -6 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 \end{array}\right]$$

$$\mathrm{Sol}(A,b) = \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 6 \\ -1 \\ 2 \\ 1 \end{pmatrix} : t \in \mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \right\} + \left\{ t \begin{pmatrix} 6 \\ -1 \\ 2 \\ 1 \end{pmatrix} : t \in \mathbb{R} \right\}$$

in-class

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -6 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 \end{array}\right]$$

$$\mathrm{Sol}(A,b) = \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 6 \\ -1 \\ 2 \\ 1 \end{pmatrix} : t \in \mathbb{R} \right\}$$

$$= \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \right\} + \left\{ t \begin{pmatrix} 6 \\ -1 \\ 2 \\ 1 \end{pmatrix} : t \in \mathbb{R} \right\}$$

$$= \{\text{partikuläre Lösung}\} + N(A)$$

in-class

1. **Solving linear systems (in-class)** (★☆☆)

   Consider the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

b) Write down a basis for $\mathbf{N}(A)$ (you might have already found it in the previous subtask), and also find a basis for $\mathbf{C}(A)$.



in-class

1. **Solving linear systems (in-class)** (★☆☆)

   Consider the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

b) Write down a basis for $\mathbf{N}(A)$ (you might have already found it in the previous subtask), and also find a basis for $\mathbf{C}(A)$.

$$\left\{ \begin{pmatrix} 6 \\ -1 \\ 2 \\ 1 \end{pmatrix} \right\}$$

in-class

1. Solving linear systems (in-class) (★☆☆)

Consider the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

b) Write down a basis for $\mathbf{N}(A)$ (you might have already found it in the previous subtask), and also find a basis for $\mathbf{C}(A)$.

$$\left\{\begin{pmatrix} 6 \\ -1 \\ 2 \\ 1 \end{pmatrix}\right\} \qquad \left\{\begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ -14 \end{pmatrix}, \begin{pmatrix} 5 \\ 12 \\ -7 \end{pmatrix}\right\}$$

in-class

1. Solving linear systems (in-class) (★☆☆)

Consider the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

c) What are the dimensions of $\mathbf{N}(A)$, $\mathbf{C}(A)$, and $\mathbf{R}(A)$?



in-class

1. Solving linear systems (in-class) (★☆☆)

Consider the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

c) What are the dimensions of $\mathbf{N}(A)$, $\mathbf{C}(A)$, and $\mathbf{R}(A)$?

1 3 3



in-class

1. **Solving linear systems (in-class)** (★☆☆)

   Consider the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

d) Determine a basis of $\mathbf{R}(A)$.

in-class

1. Solving linear systems (in-class) (★☆☆)

Consider the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

d) Determine a basis of $\mathbf{R}(A)$.

$$R(A) \quad = \quad R(R) \quad = \quad R(R')$$



in-class

1. Solving linear systems (in-class) (★☆☆)

Consider the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

d) Determine a basis of $\mathbf{R}(A)$.

$$R(A) = R(R) = R(R')$$

R(R) = R(R′) ?
yes :D

$$R = \begin{bmatrix} R' & r \\ \hline 0 & m-r \end{bmatrix}$$

in-class

1. Solving linear systems (in-class) (★☆☆)

Consider the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

d) Determine a basis of $\mathbf{R}(A)$.

$$R(A) = R(R) = R(R')$$

M invertierbar
M ist m×m, M^{-1} existiert

bzw. · M^{-1} — Zeilen werden gemischt

Mb

Lemma 3.5
R(A) = R(MA)
gleicher Zeilenraum

in-class

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -6 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 \end{array}\right]$$

in-class

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -6 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 \end{array}\right]$$

$$\left\{\begin{pmatrix} 1 \\ 0 \\ 0 \\ -6 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ -2 \end{pmatrix}\right\}$$

in-class

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & -6 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 \end{array}\right]$$

$$\left\{\begin{pmatrix} 1 \\ 0 \\ 0 \\ -6 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \\ -2 \end{pmatrix}\right\}$$

Unterschied row space & column space !!!

in-class

$A \in \mathbb{R}^{m \times n}, \quad M \in \mathbb{R}^{m \times m}$ invertierbar

	Span bleibt gleich	Positionen lin. unabhängiger Vektoren bleiben gleich
Spalten		
Reihen		

in-class

$A \in \mathbb{R}^{m \times n}, \quad M \in \mathbb{R}^{m \times m}$ invertierbar

	Span bleibt gleich	Positionen lin. unabhängiger Vektoren bleiben gleich
Spalten	Nein	
Reihen	Ja	

in-class

$A \in \mathbb{R}^{m \times n}, \quad M \in \mathbb{R}^{m \times m}$ invertierbar

A vs MA

	Span bleibt gleich	Positionen lin. unabhängiger Vektoren bleiben gleich
Spalten	Nein	Ja
Reihen	Ja	Nein

in-class

- allgemeine Lösungen für lineare Gleichungssysteme:

in-class

- allgemeine Lösungen für lineare Gleichungssysteme:
  - quadratisch, umkehrbar:
    Gauss-Elimination
  - allgemein:
    Gauss-Jordan-Elimination

$$\left[\begin{array}{ccc|c} 2 & 3 & 2 & 1 \\ 2 & 6 & 3 & 2 \end{array}\right] \xrightarrow{II \leftarrow II - I} \left[\begin{array}{ccc|c} 2 & 3 & 2 & 1 \\ 0 & 3 & 1 & 1 \end{array}\right]$$

$$\xrightarrow{I \leftarrow I - II} \left[\begin{array}{ccc|c} 2 & 0 & 1 & 0 \\ 0 & 3 & 1 & 1 \end{array}\right]$$

$$\xrightarrow[II \leftarrow \frac{1}{3} II]{I \leftarrow \frac{1}{2} I} \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \end{array}\right]$$

in-class

- allgemeine Lösungen für lineare Gleichungssysteme:
  - quadratisch, umkehrbar:
    Gauss-Elimination
  - allgemein:
    Gauss-Jordan-Elimination
    “Gauss-Elimination für nicht-quadratische Matrizen”

$$\left[\begin{array}{ccc|c} 2 & 3 & 2 & 1 \\ 2 & 6 & 3 & 2 \end{array}\right] \xrightarrow{II \leftarrow II - I} \left[\begin{array}{ccc|c} 2 & 3 & 2 & 1 \\ 0 & 3 & 1 & 1 \end{array}\right] \xrightarrow{I \leftarrow I - II} \left[\begin{array}{ccc|c} 2 & 0 & 1 & 0 \\ 0 & 3 & 1 & 1 \end{array}\right] \xrightarrow{\substack{I \leftarrow \frac{1}{2} I \\ II \leftarrow \frac{1}{3} II}} \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{3} & \frac{1}{3} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 2 & 1 \\ 2 & 6 & 3 & 2 \end{array}\right] \xrightarrow{II \leftarrow II - I} \left[\begin{array}{ccc|c} 2 & 3 & 2 & 1 \\ 0 & 3 & 1 & 1 \end{array}\right] \xrightarrow{I \leftarrow I - II} \left[\begin{array}{ccc|c} 2 & 0 & 1 & 0 \\ 0 & 3 & 1 & 1 \end{array}\right]$$

Vektorräume

ein kurzer Blick auf FS21 ...

/20

1. Gauss-Elimination Gegeben ist die Matrix $\mathbf{A}$ und der Vektor $\mathbf{b}$

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 & 5 \\ 0 & 3 & 0 & 3 \\ 0 & 0 & 4 & 8 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 \\ 6 \\ -4 \end{pmatrix}.$$

a) Der Vektor $\mathbf{x} = \begin{pmatrix} 0 & 2 & -1 & 0 \end{pmatrix}^\top$ ist eine partikuläre Lösung der Gleichung $\mathbf{Ax} = \mathbf{b}$. Bestimmen Sie die allgemeine Lösung dieser Gleichung. /10

b) Bestimmen Sie eine Basis des Bildes $\mathsf{im}(\mathbf{A})$. Begründen Sie ihre Antwort. /5

c) Gibt es Vektoren $\mathbf{c} \in \mathbb{R}^4$, für die $\mathbf{A}^\top \mathbf{x} = \mathbf{c}$ eindeutig lösbar ist? Geben Sie entweder ein Beispiel an oder begründen Sie warum es keinen solchen Vektor gibt. /5

Vektorräume

I looove this one :D (HS24)

1. Consider an arbitrary matrix $A \in \mathbb{R}^{3\times 3}$ satisfying $A\mathbf{u} = A\mathbf{v}$ for two linearly independent vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^3$. Which of the following statements must be **true**? ***Note: only one answer is correct.***

(**a**) A has rank 0.

(**b**) A has rank at least 1.

(**c**) A has rank at most 2.

(**d**) A has rank 3.

Vektorräume

time for a teeny tiny kahoot :D

Orthogonalität

$$N(A)^{\perp} = R(A) = C(A^{\top})$$

Orthogonalität

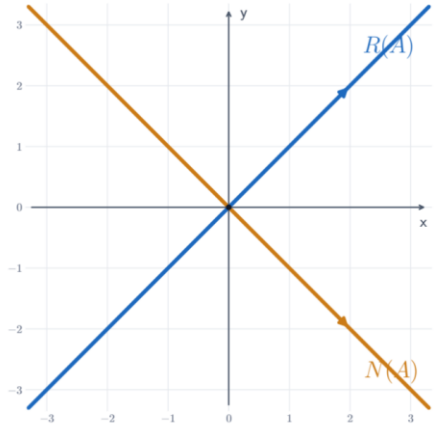
$$N(A)^{\perp} = R(A) = C(A^{\top})$$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$R(A) = \text{Span}\left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right\}$$

$$N(A) = \text{Span}\left\{\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right\}$$

$$N(A) = R(A)^{\perp}$$



Orthogonalität

remember this? :D
it all makes sense now!

Brain Teaser

Was gilt noch für A = CR'?

Eine Aussage haben wir heute benutzt. Die anderen beweisen wir später. Wieso könnten sie gelten?

Spaltenraum

$$\mathcal{C}(A) = \mathcal{C}(C)$$

✓ heute benutzt

Zeilenraum

$$\mathcal{R}(A) = \mathcal{R}(R')$$

? Beweis folgt

Nullraum

$$\mathcal{N}(A) = \mathcal{N}(R')$$

? Beweis folgt

Nullraum der Transponierten

$$\mathcal{N}(A^T) = \mathcal{N}(C^T)$$

? Beweis folgt



Orthogonalität

recap: wichtigste Theoreme zur Orthogonalität

Orthogonalität

recap: wichtigste Theoreme zur Orthogonalität

neuer Beweis für:

dim(N(A)) + dim(R(A)) = n :D

Orthogonalität

time for another (actually) teeny tiny mini kahoot! :D

bis nächste Woche!



https://linalg.guru

Zusatzslides!

true or false? (aus FS23)

Sei V ein Vektorraum über $\mathbb{R}$ mit Skalarprodukt $\langle \cdot, \cdot \rangle$ und sei $F : V \to V$ eine lineare Abbildung. Falls gilt, dass $\forall v \in V, \ \langle v, F(v) \rangle = 0$, dann ist F notwendigerweise die Nullabbildung: $F(v) = \mathbf{0}$ für alle $v \in V$.

false!



Zusatzslides!

HS24

c) Let $n \in \mathbb{N}^+$ be arbitrary and consider two arbitrary subspaces $U, V \subseteq \mathbb{R}^n$ with $\dim(U) + \dim(V) \geq n + 1$. Prove that $|U \cap V| \geq 2$.

/3 ·3

Zusatzslides!

b) Write down the inverse A^{-1} of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \in \mathbb{R}^{4\times 4}.$$

/1 ·7

Remark: Recall that the inverse can be found by solving the linear system $AX = I$ for $X \in \mathbb{R}^{4\times 4}$. Alternatively, you can also approach this as four linear systems $A\mathbf{x}_1 = \mathbf{e}_1$, $A\mathbf{x}_2 = \mathbf{e}_2$, $A\mathbf{x}_3 = \mathbf{e}_3$, and $A\mathbf{x}_4 = \mathbf{e}_4$, where $\mathbf{x}_1, \ldots, \mathbf{x}_4$ are the columns of X and $\mathbf{e}_1, \ldots, \mathbf{e}_4$ are the standard unit vectors (columns of I).

Zusatzslides!

b) Write down the inverse A^{-1} of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \in \mathbb{R}^{4\times 4}.$$

$$A^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{pmatrix}$$

/1 ·7

Remark: Recall that the inverse can be found by solving the linear system $AX = I$ for $X \in \mathbb{R}^{4\times 4}$. Alternatively, you can also approach this as four linear systems $A\mathbf{x}_1 = \mathbf{e}_1$, $A\mathbf{x}_2 = \mathbf{e}_2$, $A\mathbf{x}_3 = \mathbf{e}_3$, and $A\mathbf{x}_4 = \mathbf{e}_4$, where $\mathbf{x}_1, \ldots, \mathbf{x}_4$ are the columns of X and $\mathbf{e}_1, \ldots, \mathbf{e}_4$ are the standard unit vectors (columns of I).

Zusatzslides!

MC2. [**2 Points**] Let $A \in \mathbb{R}^{2\times 2}$ be a matrix satisfying

$$A\begin{pmatrix} x \\ y \end{pmatrix} = A\begin{pmatrix} y \\ x \end{pmatrix} \qquad \text{for any } x, y \in \mathbb{R}.$$

Which of the following statements must be **true**?

- ☐ A is symmetric.
- ☐ $A = 0$.
- ☐ $\operatorname{rank}(A) < 2$.



Zusatzslides!

MC2. [**2 Points**] Let $A \in \mathbb{R}^{2\times 2}$ be a matrix satisfying

$$A\begin{pmatrix} x \\ y \end{pmatrix} = A\begin{pmatrix} y \\ x \end{pmatrix} \qquad \text{for any } x, y \in \mathbb{R}.$$

Which of the following statements must be **true**?

- ☐ A is symmetric.
- ☐ $A = 0$.
- ☐ $\operatorname{rank}(A) < 2$.

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

Zusatzslides!

true or false? (aus FS23)

Sei V ein Vektorraum über $\mathbb{R}$ mit Skalarprodukt $\langle \cdot, \cdot \rangle$ und sei $F : V \to V$ eine lineare Abbildung. Falls gilt, dass $\forall v \in V, \ \langle v, F(v) \rangle = 0$, dann ist F notwendigerweise die Nullabbildung: $F(v) = \mathbf{0}$ für alle $v \in V$.

