Heute

Quiz (Timer)

Kahoot

Recap Orthogonal Subspaces

In-Class

1. Solving linear systems (in-class) (★☆☆)

Consider the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

a) Determine the set of solutions $\mathbf{Sol}(A, \mathbf{b}) = \{\mathbf{x} \in \mathbb{R}^4 : A\mathbf{x} = \mathbf{b}\}$, i.e., write down an *explicit* characterization of this set of solutions in the form presented after Theorem 4.42 on page 151 in the lecture notes.

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & -5 & 2 \\ 0 & -3 & -3 & 3 \\ 0 & -12 & -2 & -8 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -3 & 0 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 10 & -20 \end{bmatrix}$$
$$\to \begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} =: R.$$

$$\mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix} \to \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} \to \begin{pmatrix} 4 \\ -1 \\ -10 \end{pmatrix} \to \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} =: \mathbf{c}.$$

1. Solving linear systems (in-class) (★☆☆)

$$\begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} =: R.$$

n $A\mathbf{x} = \mathbf{b}$ with

$$= \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

a) Determine the set of solutions $\mathbf{Sol}(A, \mathbf{b}) = \{\mathbf{x} \in \mathbb{R}^4 : A\mathbf{x} = \mathbf{b}\}$, i.e., write down an *explicit* characterization of this set of solutions in the form presented after Theorem 4.42 on page 151 in the lecture notes.

b) Write down a basis for $\mathbf{N}(A)$ (you might have already found it in the previous subtask), and also find a basis for $\mathbf{C}(A)$.

1. Solving linear systems (in-class) (★☆☆)

$$\begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} =: R.$$

em $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

a) Determine the set of solutions $\mathbf{Sol}(A, \mathbf{b}) = \{\mathbf{x} \in \mathbb{R}^4 : A\mathbf{x} = \mathbf{b}\}$, i.e., write down an *explicit* characterization of this set of solutions in the form presented after Theorem 4.42 on page 151 in the lecture notes.

b) Write down a basis for $\mathbf{N}(A)$ (you might have already found it in the previous subtask), and also find a basis for $\mathbf{C}(A)$.

c) What are the dimensions of $\mathbf{N}(A)$, $\mathbf{C}(A)$, and $\mathbf{R}(A)$?

1. Solving linear systems (in-class) (★☆☆)

$$\begin{bmatrix} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix} =: R.$$

… $A\mathbf{x} = \mathbf{b}$ with

$$= \begin{bmatrix} -1 & 2 & 5 & -2 \\ -3 & 3 & 12 & -3 \\ 1 & -14 & -7 & -6 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{pmatrix} -6 \\ -15 \\ 8 \end{pmatrix}.$$

a) Determine the set of solutions $\mathbf{Sol}(A, \mathbf{b}) = \{\mathbf{x} \in \mathbb{R}^4 : A\mathbf{x} = \mathbf{b}\}$, i.e., write down an *explicit* characterization of this set of solutions in the form presented after Theorem 4.42 on page 151 in the lecture notes.

b) Write down a basis for $\mathbf{N}(A)$ (you might have already found it in the previous subtask), and also find a basis for $\mathbf{C}(A)$.

c) What are the dimensions of $\mathbf{N}(A)$, $\mathbf{C}(A)$, and $\mathbf{R}(A)$?

d) Determine a basis of $\mathbf{R}(A)$.

Exam Exercises

c) Let $n \in \mathbb{N}^+$ be arbitrary and consider two arbitrary subspaces $U, V \subseteq \mathbb{R}^n$ with $\dim(U) + \dim(V) \geq n + 1$. Prove that $|U \cap V| \geq 2$.

/3 ·3

HS24

MC5. **[2 Points]** Let $S \subseteq \mathbb{R}^n$ be a subspace and $P \in \mathbb{R}^{n \times n}$ the projection matrix onto S. Consider vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ and their projections $P\mathbf{u}$, $P\mathbf{v}$ onto S. Which of the following statements must be **true**?

- ☐ If $\mathbf{u}, \mathbf{v}$ are linearly independent, then $P\mathbf{u}, P\mathbf{v}$ are linearly independent.
- ☐ If $P\mathbf{u}, P\mathbf{v}$ are linearly independent, then $\mathbf{u}, \mathbf{v}$ are linearly independent.
- ☐ If $\mathbf{u}, \mathbf{v}$ are orthogonal, then $P\mathbf{u}, P\mathbf{v}$ are orthogonal.
- ☐ If $P\mathbf{u}, P\mathbf{v}$ are orthogonal, then $\mathbf{u}, \mathbf{v}$ are orthogonal.

Extra Exercises

Is there a matrix A such that

$$\begin{pmatrix}1\\2\\3\end{pmatrix} \in C(A^\top) \text{ (row space)} \qquad \text{and} \qquad \begin{pmatrix}3\\2\\1\end{pmatrix} \in N(A) \text{ (nullspace)}\,?$$

Construct one, or prove that none exists.

(a) Find a matrix $A \in \mathbb{R}^{2\times 2}$ with $C(A) = N(A)$.

(a) Find a matrix $A \in \mathbb{R}^{2\times 2}$ with $C(A) = N(A)$.

(b) Show that no matrix $A \in \mathbb{R}^{3\times 3}$ satisfies $C(A) = N(A)$.