Woche 6

Plan

- Recap
- In-class & Aufgaben
- Kahoot

Quiz korrigiere ich heute nachmittag!

Recap Gauss-Jordan

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix}}_{\text{Gauss (REF)}} \longrightarrow \quad ?$$


$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix}$
Gauss (REF)
$\longrightarrow$
$\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 4 \end{bmatrix}$
Gauss-Jordan (RREF)

a) Find the general solution to the system

$$\underbrace{\begin{bmatrix} 1 & 2 & -4 & 3 \\ 0 & 1 & -3 & 2 \\ 0 & 2 & -6 & 4 \end{bmatrix}}_{A} x = \underbrace{\begin{bmatrix} 11 \\ 4 \\ 8 \end{bmatrix}}_{b}$$

a) Find the general solution to the system

$$\underbrace{\begin{bmatrix} 1 & 2 & -4 & 3 \\ 0 & 1 & -3 & 2 \\ 0 & 2 & -6 & 4 \end{bmatrix}}_{A} x = \underbrace{\begin{bmatrix} 11 \\ 4 \\ 8 \end{bmatrix}}_{b}$$

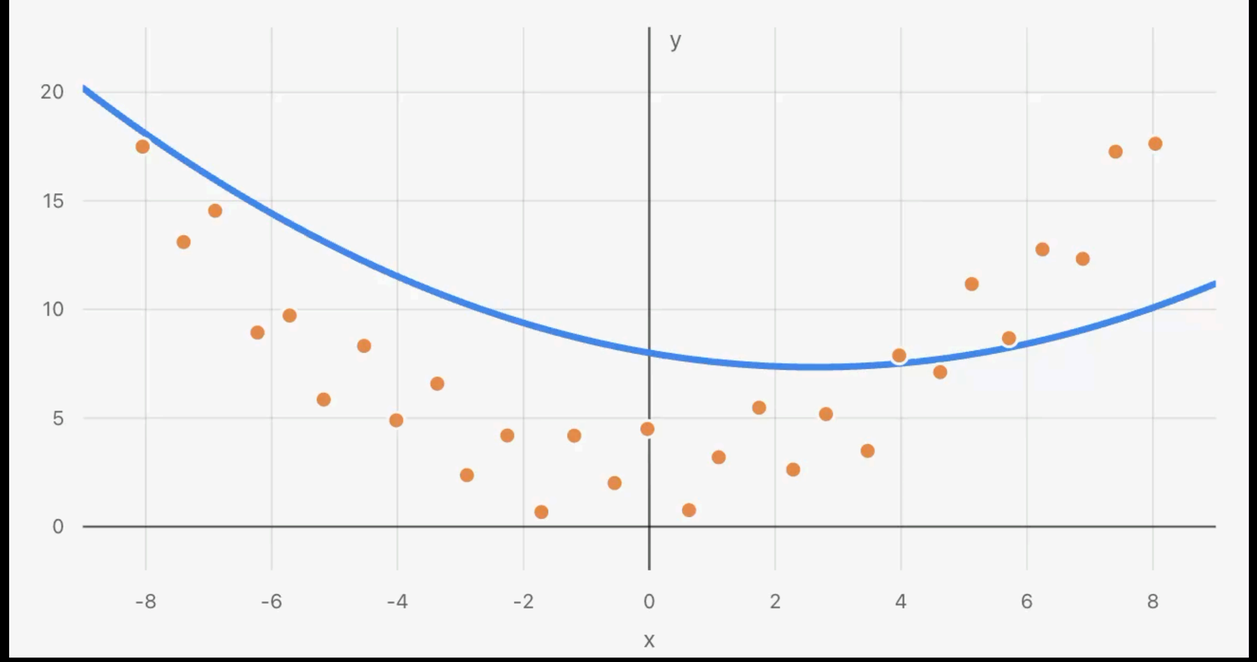
b) Find the CR decomposition for A.

Find the inverse of the following matrix using Gauss-Jordan elimination:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Recap Vector Spaces

1.	$\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v}$	*commutativity*
2.	$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$	*associativity*
3.	*There is a vector* $\mathbf{0}$ *such that* $\mathbf{v} + \mathbf{0} = \mathbf{v}$ *for all* $\mathbf{v}$	*zero vector*
4.	*There is a vector* $-\mathbf{v}$ *such that* $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$	*negative vector*
5.	$1 \cdot \mathbf{v} = \mathbf{v}$	*identity element*
6.	$(\lambda \cdot \mu)\mathbf{v} = \lambda \cdot (\mu \cdot \mathbf{v})$	*compatibility of* $\cdot$ *and* $\cdot$ *in* $\mathbb{R}$
7.	$\lambda(\mathbf{v} + \mathbf{w}) = \lambda\mathbf{v} + \lambda\mathbf{w}$	*distributivity over* $+$
8.	$(\lambda + \mu)\mathbf{v} = \lambda\mathbf{v} + \mu\mathbf{v}$	*distributivity over* $+$ *in* $\mathbb{R}$


y
20
15
10
5
0
-8
-6
-4
-2
0
2
4
6
8
x

In-Class

a) In this exercise we consider the vector space V of all real-valued functions on the interval $[0, 1]$. In other words, every element $\mathbf{f} \in V$ is a function $\mathbf{f} : [0, 1] \to \mathbb{R}$ and conversely, every function $\mathbf{f} : [0, 1] \to \mathbb{R}$ is in V. Note that it might not be obvious that this is a vector space, but for the purpose of this exercise you can assume that it is. In particular, there exists a valid addition $\mathbf{f} + \mathbf{g}$ of such functions $\mathbf{f} \in V$ and $\mathbf{g} \in V$, and a valid scalar multiplication $c\mathbf{f}$ for a scalar $c \in \mathbb{R}$ and $\mathbf{f} \in V$ defined as follows:

$$\begin{aligned} (\mathbf{f} + \mathbf{g})(x) &:= \mathbf{f}(x) + \mathbf{g}(x) && \text{for all } \mathbf{f}, \mathbf{g} \in V \text{ and } x \in [0, 1] \\ (c\mathbf{f})(x) &:= c\mathbf{f}(x) && \text{for all } \mathbf{f} \in V, x \in [0, 1] \text{ and } c \in \mathbb{R}. \end{aligned}$$

Prove that

$$U = \{\mathbf{f} \in V : \mathbf{f}(x) = \mathbf{f}(1 - x) \text{ for all } x \in [0, 1]\} \subseteq V$$

is a subspace of V.

b) Let $m \in \mathbb{N}^+$. Consider the set $\mathcal{D}_m$ of diagonal $m \times m$ matrices, which is a subspace of $\mathbb{R}^{m \times m}$. What is the dimension of $\mathcal{D}_m$? Justify your answer with a proof.

Alte Klausuraufgaben

b) Let $n \in \mathbb{N}^+$ be arbitrary. Consider two subspaces $U, V \subseteq \mathbb{R}^n$ and assume that there exists a vector $u \in U \setminus V$ and a vector $v \in V \setminus U$. Prove that $U \cup V$ is not a subspace.

/3 ·2

Weitere Aufgaben

Let $n \in \mathbb{N}^+$ be arbitrary. Consider two subspaces $U, V \subseteq \mathbb{R}^n$. Prove that $U \cap V$ is a subspace of $\mathbb{R}^n$.

Let $n \in \mathbb{N}^+$ be arbitrary. Consider two subspaces $U, V \subseteq \mathbb{R}^n$. Prove that $U \setminus V$ is not a subspace of $\mathbb{R}^n$.