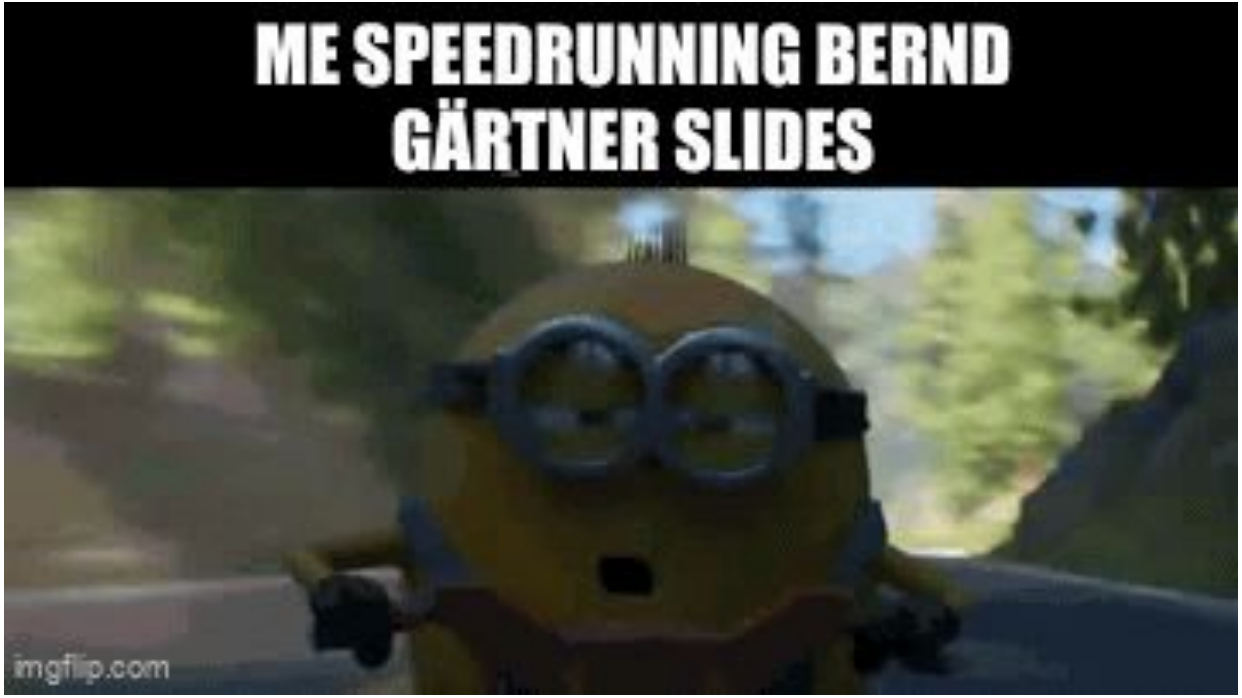
# Heute



# Gauss-Jordan

$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix}}_{\text{Gauss (REF)}} \longrightarrow ?$$

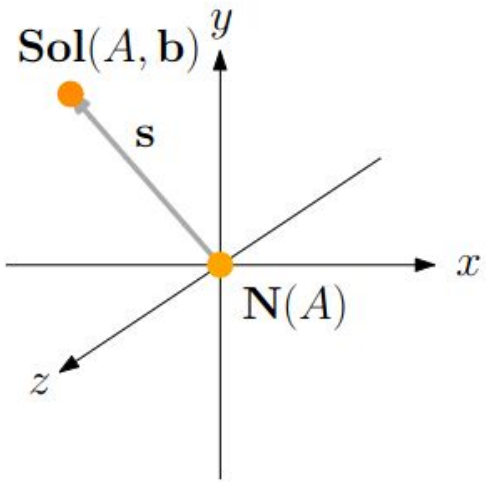
$$\underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix}}_{\text{Gauss (REF)}} \longrightarrow \underbrace{\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 4 \end{bmatrix}}_{\text{Gauss-Jordan (RREF)}}$$

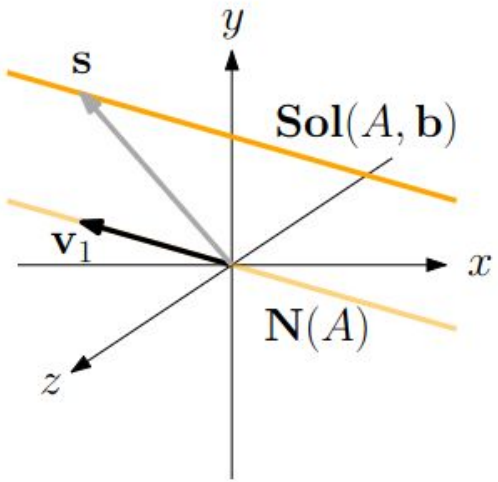
# Gauss-Jordan -> CR

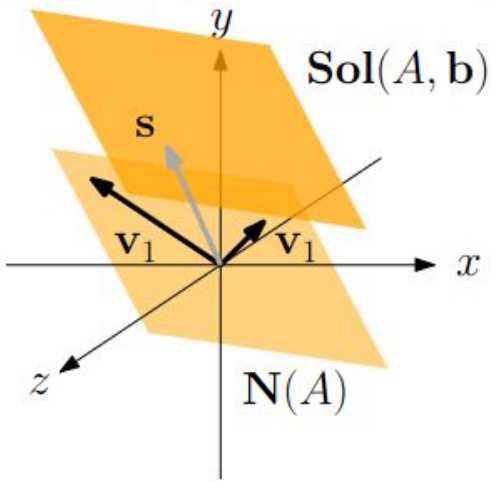
# Gauss-Jordan -> Nullspace

# General Solution

**Theorem 4.42**: Let $\mathbf{s}$ be a solution of $A\mathbf{x} = \mathbf{b}$. Then $\mathbf{Sol}(A, \mathbf{b}) = \{\mathbf{s} + \mathbf{x} : \mathbf{x} \in \mathbf{N}(A)\}$.



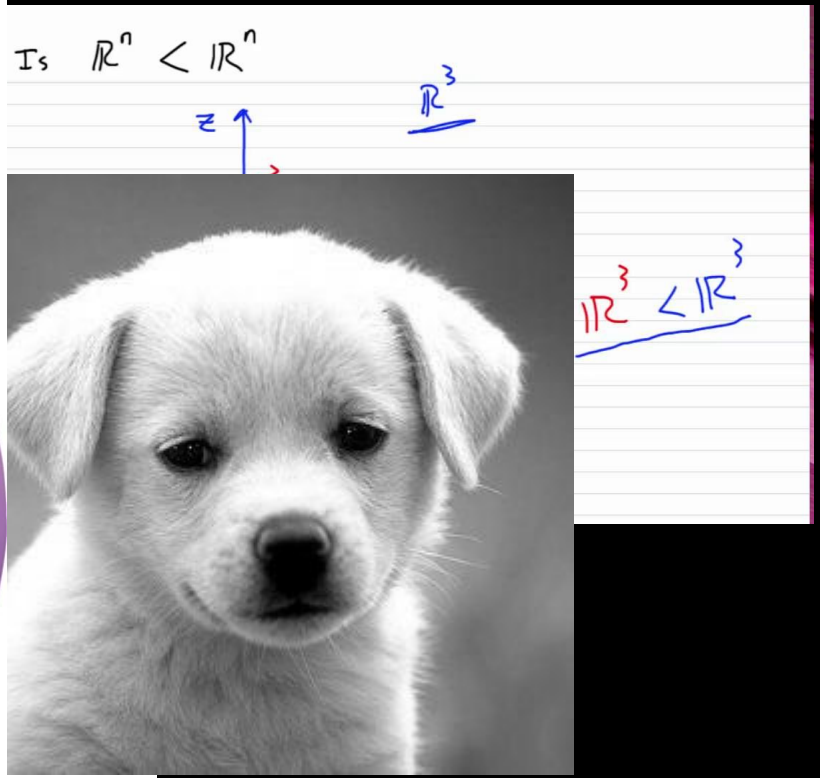

Which rank is each picture?

# Intro Vector Spaces, Subspaces

| | | |
|---|---|---|
| 1. | $\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v}$ | *commutativity* |
| 2. | $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$ | *associativity* |
| 3. | *There is a vector* $\mathbf{0}$ *such that* $\mathbf{v} + \mathbf{0} = \mathbf{v}$ *for all* $\mathbf{v}$ | *zero vector* |
| 4. | *There is a vector* $-\mathbf{v}$ *such that* $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$ | *negative vector* |
| 5. | $1 \cdot \mathbf{v} = \mathbf{v}$ | *identity element* |
| 6. | $(\lambda \cdot \mu)\mathbf{v} = \lambda \cdot (\mu \cdot \mathbf{v})$ | *compatibility of* $\cdot$ *and* $\cdot$ *in* $\mathbb{R}$ |
| 7. | $\lambda(\mathbf{v} + \mathbf{w}) = \lambda\mathbf{v} + \lambda\mathbf{w}$ | *distributivity over* $+$ |
| 8. | $(\lambda + \mu)\mathbf{v} = \lambda\mathbf{v} + \mu\mathbf{v}$ | *distributivity over* $+$ *in* $\mathbb{R}$ |

# Vector Spaces:



# Kahoot

# In-Class

**a)** In this exercise we consider the vector space $V$ of all real-valued functions on the interval $[0, 1]$. In other words, every element $\mathbf{f} \in V$ is a function $\mathbf{f} : [0, 1] \to \mathbb{R}$ and conversely, every function $\mathbf{f} : [0, 1] \to \mathbb{R}$ is in $V$. Note that it might not be obvious that this is a vector space, but for the purpose of this exercise you can assume that it is. In particular, there exists a valid addition $\mathbf{f} + \mathbf{g}$ of such functions $\mathbf{f} \in V$ and $\mathbf{g} \in V$, and a valid scalar multiplication $c\mathbf{f}$ for a scalar $c \in \mathbb{R}$ and $\mathbf{f} \in V$ defined as follows:

$$(\mathbf{f} + \mathbf{g})(x) := \mathbf{f}(x) + \mathbf{g}(x) \qquad \text{for all } \mathbf{f}, \mathbf{g} \in V \text{ and } x \in [0, 1]$$
$$(c\mathbf{f})(x) := c\mathbf{f}(x) \qquad \text{for all } \mathbf{f} \in V, x \in [0, 1] \text{ and } c \in \mathbb{R}.$$

Prove that

$$U = \{\mathbf{f} \in V : \mathbf{f}(x) = \mathbf{f}(1 - x) \text{ for all } x \in [0, 1]\} \subseteq V$$

is a subspace of $V$.

**b)** Let $m \in \mathbb{N}^+$. Consider the set $\mathcal{D}_m$ of diagonal $m \times m$ matrices, which is a subspace of $\mathbb{R}^{m \times m}$. What is the dimension of $\mathcal{D}_m$? Justify your answer with a proof.

# Extra Exercises

**Lemma 4.9:** For every vector $\mathbf{v}$, we have $0\mathbf{v} = \mathbf{0}$.

Use:

| | | |
|---|---|---|
| 1. | $\mathbf{v} + \mathbf{w} = \mathbf{w} + \mathbf{v}$ | *commutativity* |
| 2. | $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$ | *associativity* |
| 3. | *There is a vector* $\mathbf{0}$ *such that* $\mathbf{v} + \mathbf{0} = \mathbf{v}$ *for all* $\mathbf{v}$ | *zero vector* |
| 4. | *There is a vector* $-\mathbf{v}$ *such that* $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$ | *negative vector* |
| 5. | $1 \cdot \mathbf{v} = \mathbf{v}$ | *identity element* |
| 6. | $(\lambda \cdot \mu)\mathbf{v} = \lambda \cdot (\mu \cdot \mathbf{v})$ | *compatibility of* $\cdot$ *and* $\cdot$ *in* $\mathbb{R}$ |
| 7. | $\lambda(\mathbf{v} + \mathbf{w}) = \lambda\mathbf{v} + \lambda\mathbf{w}$ | *distributivity over* $+$ |
| 8. | $(\lambda + \mu)\mathbf{v} = \lambda\mathbf{v} + \mu\mathbf{v}$ | *distributivity over* $+$ *in* $\mathbb{R}$ |

# Are the following sets subspaces?

$\varnothing$

$\{\mathbf{0}\}$

Let $n \in \mathbb{N}^+$ be arbitrary. Consider two subspaces $U, V \subseteq \mathbb{R}^n$. Prove that $U \setminus V$ is not a subspace of $\mathbb{R}^n$.

Let $n \in \mathbb{N}^+$ be arbitrary. Consider two subspaces $U, V \subseteq \mathbb{R}^n$. Prove that $U \cap V$ is a subspace of $\mathbb{R}^n$.

# Exam Exercises

**b)** Let $n \in \mathbb{N}^+$ be arbitrary. Consider two subspaces $U, V \subseteq \mathbb{R}^n$ and assume that there exists a vector $u \in U \setminus V$ and a vector $v \in V \setminus U$. Prove that $U \cup V$ is not a subspace.

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