

In-Class

1. Interpolation (in-class (★☆☆)) Assume that you have collected the following data points

x	y
0	1
2	2
4	5
6	6

and you want to find a function $f : \mathbb{R} \rightarrow \mathbb{R}$ that interpolates them, i.e. f should satisfy $f(x) = y$ for all pairs of x, y given by the table above. There is an abundance of functions that you can try and in particular, there are many different functions that do interpolate the four datapoints. In this exercise, we are interested in polynomials, i.e. we restrict f to be a polynomial of degree at most 3. In particular, this means that f has the form $f(x) = ax^3 + bx^2 + cx + d$ for some $a, b, c, d \in \mathbb{R}$. Your task is to find values for a, b, c, d such that f interpolates all four points given in the table.

Proof Lemma 3.5

$$\mathbf{v} \in \mathbf{C}(A^\top)$$



$$\mathbf{v} \in \mathbf{C}(A^\top M^\top).$$

$$\mathbf{v} \in \mathbf{C}(A^\top)$$



$$\mathbf{v} = A^\top \mathbf{x} \text{ for some } \mathbf{x} \in \mathbb{R}^m$$



$$\mathbf{v} = A^\top M^\top \mathbf{y} \text{ for some } \mathbf{y} \in \mathbb{R}^m$$



$$\mathbf{v} \in \mathbf{C}(A^\top M^\top).$$

Gauss Finding Inverse

Exercise. Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}.$$

Compute A^{-1} .

Why those row ops?

Extra (Exam)

1. Let $A \in \mathbb{R}^{5 \times 6}$ with $\text{rank}(A) = 5$ and $\mathbf{b} \in \mathbb{R}^5$ be arbitrary. Which of the following statements must be **true**? *Note: only one answer is correct.*

- (a) The linear system $A\mathbf{x} = \mathbf{b}$ has no solutions, i.e. there is no $\mathbf{x} \in \mathbb{R}^6$ such that $A\mathbf{x} = \mathbf{b}$.
- (b) The linear system $A\mathbf{x} = \mathbf{b}$ has exactly one solution, i.e. there is exactly one $\mathbf{x} \in \mathbb{R}^6$ such that $A\mathbf{x} = \mathbf{b}$.
- (c) The linear system $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions, i.e. there exist infinitely many distinct $\mathbf{x} \in \mathbb{R}^6$ such that $A\mathbf{x} = \mathbf{b}$.
- (d) The linear system $A\mathbf{x} = \mathbf{b}$ has exactly two solutions, i.e. there are exactly two distinct choices of $\mathbf{x} \in \mathbb{R}^6$ such that $A\mathbf{x} = \mathbf{b}$.

1. Let $A \in \mathbb{R}^{5 \times 5}$ with $\sum_{j=1}^5 A_{ij} = 0$ for all $i \in \{1, 2, \dots, 5\}$. Which of the following statements must be true? *Note: only one answer is correct.*

- (a) A is the zero matrix, i.e. $A = 0$.
- (b) A has an inverse A^{-1} .
- (c) The rank of A is less than 5, i.e. $\text{rank}(A) < 5$.
- (d) There are only finitely many vectors $\mathbf{x} \in \mathbb{R}^5$ with $A\mathbf{x} = \mathbf{0}$.

Extra Exercises

Let $A \in \mathbb{R}^{m \times m}$ and $c \geq 0$ be such that

$$\sum_{j=1}^m |a_{ij}| \leq c$$

holds for all $i \in [m]$. Prove that for all $k \geq 1$ and all $i, j \in [m]$,

$$|(A^k)_{ij}| \leq c^k.$$

Extra - Extra (Exam)

MC2. [2 Points] Let $A \in \mathbb{R}^{2 \times 2}$ be a matrix satisfying

$$A \begin{pmatrix} x \\ y \end{pmatrix} = A \begin{pmatrix} y \\ x \end{pmatrix} \quad \text{for any } x, y \in \mathbb{R}.$$

Which of the following statements must be **true**?

- ☐ $\lambda = 1$ is an eigenvalue of A .
- ☐ A is symmetric.
- ☐ $A = 0$.
- ☐ $\text{rank}(A) < 2$.

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b) [3 Points] Let $A \in \mathbb{R}^{n \times n}$ be a symmetric matrix with $\text{Tr}(A^2) = 0$. Show that $A = 0$.

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