

In-Class

1. Matrix multiplication with vectors and covectors (in-class) (★☆☆)

Let $n \in \mathbb{N}$. Consider $\mathbf{v}, \mathbf{w} \in \mathbb{R}^n$ given by $\mathbf{v} = \begin{pmatrix} 1 \\ 2 \\ \vdots \\ n \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} 2 \\ 2 \\ \vdots \\ 2 \end{pmatrix}$.

a) Compute $\mathbf{v}^\top \mathbf{w}$ for all $n \in \mathbb{N}$.

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b) Compute $\mathbf{vw}^\top$ when $n = 4$.

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c) Compute $\mathbf{w}^\top (\mathbf{v} \mathbf{w}^\top) \mathbf{v}$ for all $n \in \mathbb{N}$.

2. Exercise 2.45 (in-class) (★☆☆)

Let $A \in \mathbb{R}^{m \times n}$ of rank r and $C \in \mathbb{R}^{m \times r}$ and $R' \in \mathbb{R}^{r \times n}$ be the matrices in the CR-decomposition of A as given in Theorem 2.44.

- a) Suppose $r = n$. What are the matrices C and R' ?

Let $A \in \mathbb{R}^{m \times n}$ of rank r and $C \in \mathbb{R}^{m \times r}$ and $R' \in \mathbb{R}^{r \times n}$ be the matrices in the CR-decomposition of A as given in Theorem 2.44.

b) Suppose $r = 0$. What are the matrices C and R' ?

Extra (Exam)

c) Let $n \in \mathbb{N}^+$ and let $A \in \mathbb{R}^{n \times n}$ be a matrix satisfying $A^\top + A = 2I$. Prove that $\mathbf{x}^\top A \mathbf{x} = \|\mathbf{x}\|^2$ for all $\mathbf{x} \in \mathbb{R}^n$.

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Extra Exercises

Exercise. Let $A \in \mathbb{R}^{n \times n}$ be invertible. Show that A has exactly one inverse. That is: if $B, C \in \mathbb{R}^{n \times n}$ both satisfy

$$AB = BA = I_n \quad \text{and} \quad AC = CA = I_n,$$

then $B = C$.

Lemma 2.59. *Let A be an invertible $m \times m$ matrix. Then the transpose $A^\top$ is also invertible, and*

$$(A^\top)^{-1} = (A^{-1})^\top.$$