

Lineare Algebra

KW 38



Nina Gassner

Wie funktioniert der Kurs überhaupt?

- alle wichtigsten Informationen auf:
<https://ti.inf.ethz.ch/ew/courses/LA26/index.html>

wöchentlich:

- Vorlesungen (Aufzeichnungen)
 - Übungsstunden (Donnerstag / Freitag; online)
 - Übungen (hand-in, in-class)
 - Quiz alle zwei Wochen (Bonus!)
 - Moodle-Aufgaben
 - Study Center
-
- bookmarks!
 - Zusammenhang: andere Kurse

$$\min \left(\frac{0.25}{5} \sum_{i=1}^7 p_i, 0.25 \right).$$



wofür Übungsstunden?

wissen



verstehen

nicht: Ersatz für Vorlesungen

Erwartung: Vorlesungen/Vorlesungsmaterial bereits gesehen



Wie sieht die Prüfung aus?

1. Rechenaufgaben üben!
2. Beweisaufgaben Ideen sehr nah an Übungen / Vorlesungen
3. Multiple Choice–Aufgaben

cheat sheet !



Wie sehen meine Übungsstunden aus?

- alle zwei Wochen: Quiz
- relevante Theorie / Herleitungen ansehen
- Aufgaben gemeinsam lösen nicht vorrechnen, mitdenken! (Schokolade :D)

Ziel:

- wie? Aufgabentypen erkennen, Ansätze kennen und anwenden
- wieso? Begründung verstehen, geometrische Intuition entwickeln

- Kahoot gezielte Vorbereitung auf MC der Prüfung

Wünsche äussern! :)

anderer Fokus? was fällt euch schwer? zu schnell/langsam?



viel Wert auf:

- Interaktivität
- verschiedene Blickwinkel auf Theorie:
 - geometrisch/intuitiv
 - formal/mathematisch



Wie lernt man?

- **kein richtiger/falscher Weg!**
- **Imposter-Syndrom**
- **Ihr braucht Pausen und Schlaf!**



Wie lernt man?

Wenn es stressig wird

- priorisieren
 - Überblick darüber behalten, was fehlt!
(später nachholen)
 - Kollaboration
 - am Ball bleiben!
-
- vorarbeiten falls möglich/sinnvoll

denkt daran:

- nicht erwartet, alles 100% zu verstehen
- Ihr seid nicht allein – vielen geht es so, Unterstützung vorhanden
- Lernphase, PVK
- ~40% schaffen es nicht beim ersten Versuch – und das ist völlig ok & normal!



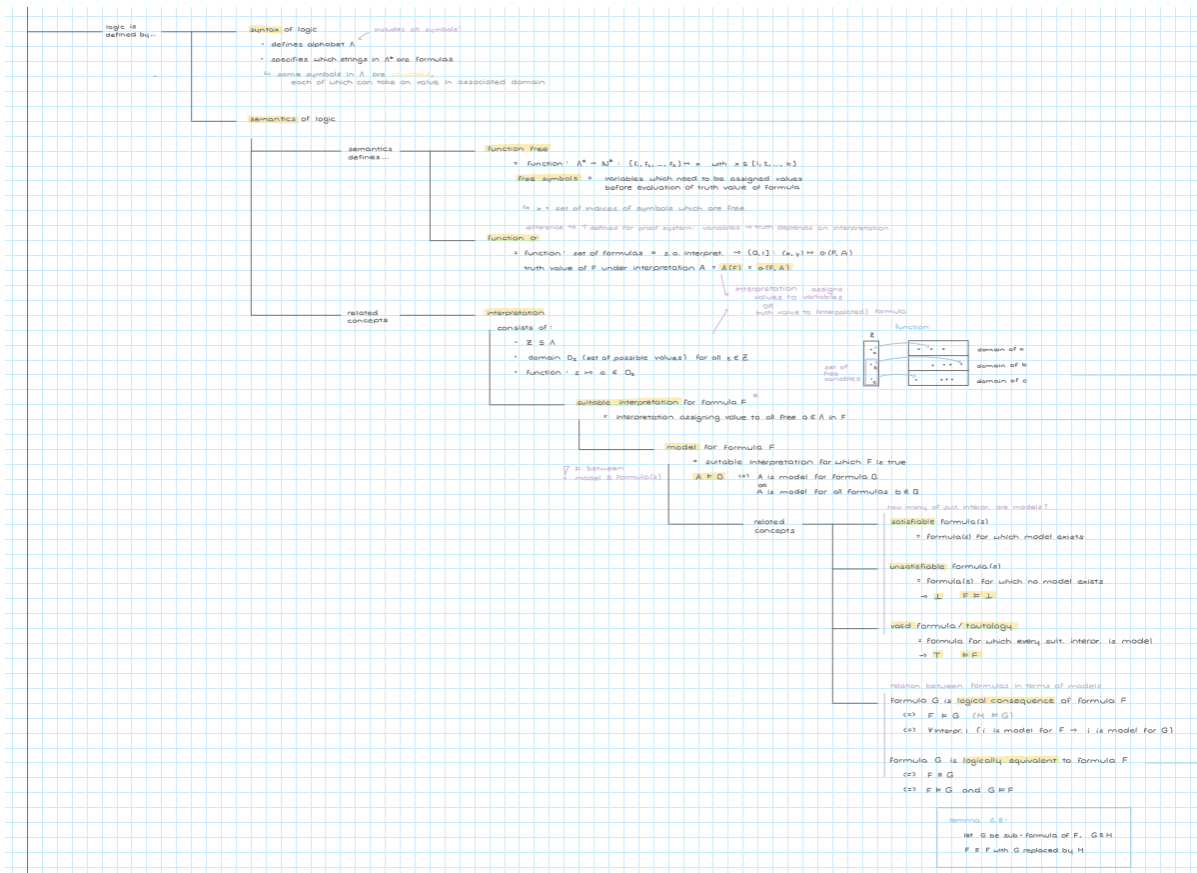
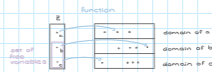
persönliche Empfehlung

- mind maps
- Gedächtnispalast
 - Zusammenhänge verbildlichen
 - active recall
 - wissen, wo man neues Wissen einfügt
 - Intuition

wichtigster Schritt, nachdem man etwas gelernt hat:

- nochmals Überblick verschaffen
- sich/anderer Person erklären
 - da klickt es!





How many of suit interior are models?

satisfiable formula(s)

- formula(s) for which model exists

unsatisfiable formula(s)

- formula(s) for which no model exists
- $\rightarrow$ $\neg$ $F \models \perp$

valid formula / **tautology**

- formula for which every suit interior is model
- $\rightarrow$ $\models$ $F \models \perp$

relation between formulae in terms of models

formula G is **logical consequence** of formula F

- $\Leftrightarrow F \models G \Leftrightarrow (H, H \models G)$
- $\Leftrightarrow \forall \text{interp. } i: i \text{ is model for } F \Rightarrow i \text{ is model for } G$

formula G is **logically equivalent** to formula F

- $\Leftrightarrow F \models G$
- $\Leftrightarrow F \models G$ and $G \models F$

lemma: Δ, Σ

let G be sub-formula of F , $G \models H$

F, F with G replaced by H



persönliche Empfehlung

3Blue1Brown



3Blue1Brown

week 00:

https://youtu.be/fNk_zzaMoSs?si=NyHtsVDPOelb8qtz

week 01:

<https://youtu.be/k7RM-ot2NWY?si=TakDx48upLLtJdOw>



Fragen?



check out our website! :D

<https://linalg.guru>



zum Stoff!



some basics first

- Was ist $\mathbb{R}^m$?
= Menge aller m-Tupel!
- Vektor-Notation



Linearkombination?

Linear combination: all in one!

Definition 1.4: Let $\mathbf{v}, \mathbf{w} \in \mathbb{R}^m$, $\lambda, \mu \in \mathbb{R}$. Then

$$\lambda \mathbf{v} + \mu \mathbf{w} \in \mathbb{R}^m$$

is a *linear combination* of $\mathbf{v}$ and $\mathbf{w}$.

If $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n \in \mathbb{R}^m$ and $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{R}$, then

$$\lambda_1 \mathbf{v}_1 + \lambda_2 \mathbf{v}_2 + \dots + \lambda_n \mathbf{v}_n$$

is a *linear combination* of $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$.

= «alles, was durch Kombinieren der gegebenen Vektoren irgendwie erreicht werden kann»



Linearkombination!

3. Geometry of linear combinations (in-class) (★☆☆)

In this exercise, you are asked to sketch sets of points in $\mathbb{R}^3$. No formal justification is required.

a) Draw the set of linear combinations $\left\{ \lambda_1 \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix} + \lambda_3 \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} : \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R} \right\}$.

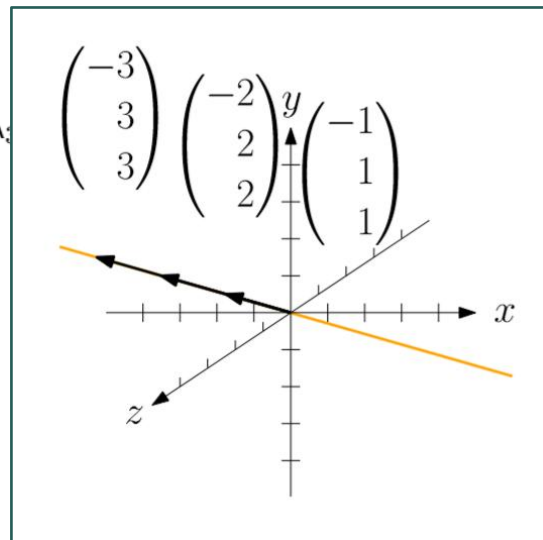


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Linearkombination!

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b) Draw the set of linear combinations $\left\{ \lambda_1 \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} + \lambda_2 \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix} : \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R} \right\}$.

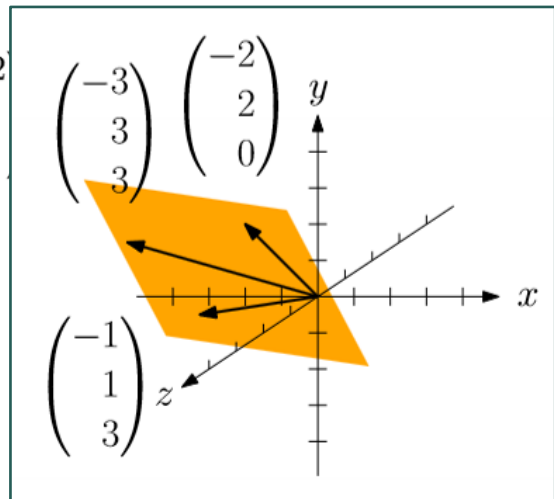


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Linearkombination!

3. Geometry of linear combinations (in-class) (★☆☆)

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c) Draw the set of linear combinations $\left\{ \lambda_1 \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix} + \lambda_2 \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} : \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R} \right\}$.

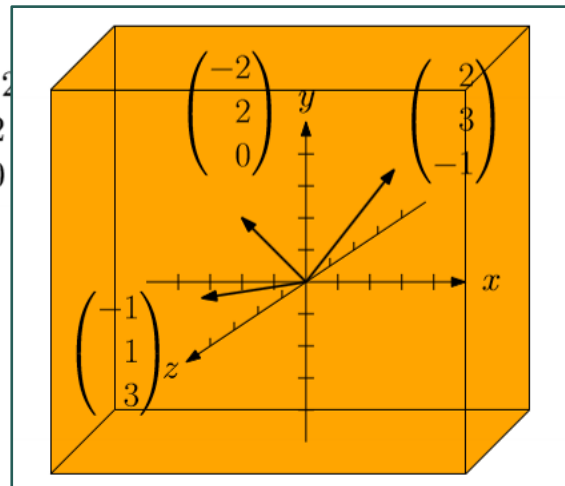


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Linearkombination!

1. Linear combinations of vectors (hand-in) (★☆☆)

- a) Prove that every vector in $\mathbb{R}^2$ can be written as a linear combination of $\mathbf{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbf{w} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

Fact 1.5: Every vector $\mathbf{u} \in \mathbb{R}^2$ is a linear combination of

$$\mathbf{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}.$$



in-class 2

2. The perfect long drink (in-class) (★☆☆)

- a) Suppose that you would like to mix the perfect long drink from the two ingredients G and T . Your sources tell you that the perfect long drink is defined as 23ml of G and 77ml of T . Unfortunately, your friends already mixed two imperfect drinks: One with 15ml of G and 85ml of T , and another one with 35ml of G and 65ml of T . How can you use the two imperfect drinks to make one perfect drink?



in-class 2

b) One could model the set of all possible 100ml drinks mixed from G and T as

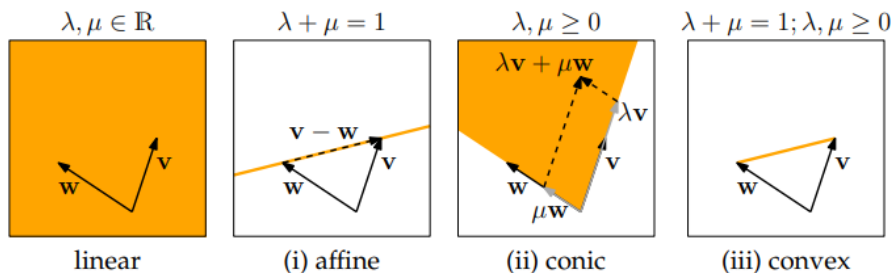
$$D := \left\{ \begin{pmatrix} g \\ t \end{pmatrix} \in \mathbb{R}^2 : g + t = 100, g \geq 0, t \geq 0 \right\}.$$

The two imperfect drinks are then represented by the vectors $\mathbf{v} = \begin{pmatrix} 15 \\ 85 \end{pmatrix} \in D$ and $\mathbf{w} = \begin{pmatrix} 35 \\ 65 \end{pmatrix} \in D$, respectively. Using this formulation, write down the set $\hat{D} \subseteq D$ of all 100ml drinks that you could mix from $\mathbf{v}$ and $\mathbf{w}$. What geometric shape does this set have?



in-class 2

Affine, conic, convex combinations: special linear combinations (Definition 1.7)



$$\begin{aligned}\lambda\mathbf{v} + \mu\mathbf{w} \\ &= \lambda\mathbf{v} + (1 - \lambda)\mathbf{w} \\ &= \mathbf{w} + \lambda(\mathbf{v} - \mathbf{w})\end{aligned}$$



in-class 2

- c) Finally, we consider the set $\overline{D}$ of drinks of any size that can be mixed from the two drinks $\mathbf{v}$ and $\mathbf{w}$. What geometric shape does $\overline{D}$ have?



Kahoot time! :D



bis nächste Woche!



<https://linalg.guru>

