b) **[3 Points]** Let $A \in \mathbb{R}^{n \times n}$ be a symmetric matrix with $\mathrm{Tr}(A^2) = 0$. Show that $A = 0$.

HS25

HS25

MC2. **[2 Points]** Let $A \in \mathbb{R}^{2\times 2}$ be a matrix satisfying

$$A\begin{pmatrix} x \\ y \end{pmatrix} = A\begin{pmatrix} y \\ x \end{pmatrix} \qquad \text{for any } x, y \in \mathbb{R}.$$

Which of the following statements must be **true**?

- ☐ $\lambda = 1$ is an eigenvalue of A.
- ☐ A is symmetric.
- ☐ $A = 0$.
- ☐ $\operatorname{rank}(A) < 2$.

Gegeben sei das lineare Gleichungssystem $\mathbf{Ax} = \mathbf{b}$ mit

$$\mathbf{A} = \begin{pmatrix} 2 & 1 & 0 \\ 2 & 3 & 2 \\ 6 & 5 & 2 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 3 \\ 7 \end{pmatrix}.$$

a) Geben Sie mithilfe des Gauss-Verfahrens die Lösungsmenge an.

FS19

HS23

b) Let $n \in \mathbb{N}^+$ and let $S \in \mathbb{R}^{n \times n}$ be a matrix satisfying $S^\top = -S$. Prove that $\mathbf{x}^\top S \mathbf{x} = 0$ for all $\mathbf{x} \in \mathbb{R}^n$.

/3 ·2

HS23

c) Note that this exercise is intended to be more **challenging** than the others! Let $n \in \mathbb{N}^+$ be arbitrary and consider the matrix $A \in \mathbb{R}^{n \times n}$ defined as

$$A := I + B$$

where $I \in \mathbb{R}^{n \times n}$ is the identity matrix and $B \in \mathbb{R}^{n \times n}$ satisfies $B_{ij} = 1$ for all $i, j \in \{1, 2, \ldots, n\}$ (i.e. all entries of B are 1). Prove that A is invertible.

/3 ·3